Module - 5
Game Theory.

Game Theory:
The term game represents a competition between two or more parties. A situation is termed as game when it posses the following properties: 1) The no of competitors is finite.
2) There is a competition between the participants
3) The rules must known to all players.
4) The outcome of the game is affected by the choices made by all the players.

Strategy: The term strategy is defined as a complete set of plans of action. The players we consider during the play of the game i.e strategy of a player is the decision rule.
Strategy can be classified as 1) pure strategy 2) mixed strategy.

Pure strategy: A strategy is called pure if all the players know the rules

Mixed strategy: The strategy is mixed strategy if the probability of combination of available choices of strategy

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Types of Games:

1) ~~2 person games~~
b) n person games

1) 2 person game & n person game:
In two person games the players may have many possible choices to them for each play of the game, but the number of players remain only two. Hence it is called two person game. In case of more than two persons, the game is generally called n person game.

2) Zero sum game:
Zero sum game is one in which the sum of the payments to all the competitors is zero, for every possible outcome of the game if sum of the points scored is equal to sum of the points lost.

3) Two person zero sum game:
The game with two players where the game of one player is equal to loss of other is known as two person zero sum game. It is also called as rectangular game.

Characteristics of 2 player zero sum game.
* Only 2 players participate in the game.
* Each player has a finite number of strategies to use.
* Total pay off to the two players at the

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end of each play is zero.

pay-off matrix

player B

$$
\text{player A}\quad
\begin{array}{c|cccccc|}
 & 1 & 2 & 3 & 4 & \cdots & m \\
\hline
1 & a_{11} & a_{12} & a_{13} & a_{14} & \cdots & a_{1m} \\
2 & a_{21} & a_{22} & a_{23} & a_{24} & \cdots & a_{2m} \\
3 & \vdots & & & & & \vdots \\
4 & \vdots & & & & & \vdots \\
\vdots & \vdots & & & & & \vdots \\
n & a_{n1} & \cdots & & & \cdots & a_{nm} \\
\hline
\end{array}
$$

player A

Ex:

$$
\text{player A}\quad
\begin{array}{c|ccc|}
 & 1 & 2 & 3 \\
\hline
1 & 4 & 5 & 6 \\
2 & -7 & -8 & 9 \\
3 & 1 & 2 & -3 \\
\hline
\end{array}
$$

Maximin - Minmax principle:

Defination:

Maximin - Minmax:

This principle is used for the selection of optimal strategies by two players. Consider two players A & B. A is a player who wsh wishes to maximize his game while player B wishes to minimize his loss. Since A player would try to maximize his minimum game we obtain for player A a value called maximin value and the corresponding strategy is called maximin strategy.

Since the player B wishes to minimize his

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loss, the value is called minimax value which is the minimum of maximum loss. The corresponding strategy is called minmax strategy.

Note: When maximin value is equal to minmax value the corresponding strategy is called optimal strategy, ~~And~~ game and game have "saddle point". The value of the game is given by "saddle point"

Saddle point: A saddle point is a position in the pay off matrix where maximum of row minima considered with minimum of column maximum. The pay off at the saddle point is called the value of the game.

1) Solve the game who's pay off matrix is given below

Player B

$$\text{Player A} \quad \begin{array}{c|cccc} & B_1 & B_2 & B_3 & B_4 \\ \hline A_1 & 1 & 3 & 1 & \\ A_2 & 0 & -4 & -3 & \\ A_3 & 1 & 5 & -1 & \\ A_4 & & & & \end{array}$$

Gain for player A is loss for player B

Step 1: Find out the row minimum & column maximum

$$\begin{array}{c} A_1 \\ A_2 \\ A_3 \\ \\ \end{array} \begin{bmatrix} \boxed{1} & 3 & \boxed{1} \\ 0 & -4 & -3 \\ 1 & 5 & -1 \end{bmatrix} \begin{array}{c} 1 \\ -4 \\ -1 \end{array}$$

$$\quad 1 \quad 5 \quad 1$$

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Step 2: find out min max

Min = {max} → minimum of maximum
= 1 among (1, 5, 1)

max = {min} → maximum of minimum
= 1 among (1, -4, -1)

∴ maxmin = minmax

1 = 1

The game has optimal strategy.
saddle point is 1. Strategy for A = A_1 & ~~A_3~~
Strategy for B = B_1 & B_3

2) Determine which of the following 2 person zero sum games are optimal strategies

a)

$$\begin{array}{c|cc} & B_1 & B_2 \\ \hline A_1 & -5 & 2 \\ A_2 & -7 & -4 \end{array}$$

b)

$$\begin{array}{c|cc} & B_1 & B_2 \\ \hline A_1 & 1 & 1 \\ A_2 & 4 & -3 \end{array}$$

a) Step 1: find out row minima & column maxima

	B_1	B_2	row min
A_1	$\boxed{-5}$	2	-5
A_2	-7	-4	-7
column max	-5	2	

Step 2: find out min max

Min = {max} = -5

max = {min} = -5

Saddle point = -5

optimal strategies O.S = [A_1 & B_1]

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b) Step1: Find row min and column max

$$\begin{array}{c|cc|c} & B_1 & B_2 & \text{row min} \\ A_1 & 1 & \boxed{1} & 1 \\ A_2 & 4 & -3 & -3 \\ \text{col max} & 4 & 1 & \end{array}$$

Step 2: Min = {max} = 1
max = {min} = 1
saddle point = 1
optimal strategies A_1, B_2 and B_2

3) Find saddle pt and value of the game

$$\begin{array}{c|ccc|} & B_1 & B_2 & B_3 \\ A_1 & 15 & 2 & 3 \\ A_2 & 6 & 5 & 7 \\ A_3 & -7 & 4 & 0 \end{array}$$

Step1: Find row min and column max

$$\begin{array}{c|ccc|c} & B_1 & B_2 & B_3 & \text{row min} \\ A_1 & 15 & 2 & 3 & 2 \\ A_2 & 6 & \boxed{5} & 7 & 5 \\ A_3 & -7 & 4 & 0 & 0 \\ \text{col max} & 15 & 5 & 7 & \end{array}$$

Step2: Min = {max} = 5
max = {min} = 5
saddle point = 5
Optimal strategies for A : A_2
for B : B_2

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	B_1	B_2	B_3	B_4
A_1	1	2	1	20
A_2	5	5	4	6
A_3	4	-2	0	-5

Step 1:

	B_1	B_2	B_3	B_4	row min
A_1	1	2	1	20	1
A_2	5	5	$\boxed{4}$	6	4
A_3	4	-2	0	-5	-5
col max	5	5	4	20	

Step 2: min = {max} = 4

max = {min} = 4

saddle point = 4

Optimal strategies for player A : A_2, ~~A_3~~

player B : ~~B_1~~, B_3

5.

	B_1	B_2	B_3	B_4	row min
A_1	1	7	3	4	1
A_2	5	6	$\boxed{4}$	5	4
A_3	7	2	0	3	0
col max	7	7	4	5	

Step 2: Min = {max} = 4

max = {min} = 4

saddle point = 4

for player A = ~~A_1~~, A_2

for player B = B_3, ~~B_4~~

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Games without saddle points means mixed Strategies

2x2 games without saddle points:

	b_1	b_2
a_1	a	b
a_2	c	d

$$p_1 = \frac{d-c}{(a+d)-(b+c)} \qquad p_2 = 1 - p_1$$

$$q_1 = \frac{d-b}{(a+d)-(b+c)} \qquad q_2 = 1 - q_1$$

$$v = \frac{ad-bc}{(a+d)-(b+c)}$$

1)

$$\begin{array}{c|cc} & B_1 & B_2 \\ \hline A_1 & 8 & -3 \\ A_2 & -3 & 1 \end{array}$$

Step 1: Check for saddle point

$$\begin{array}{c|cc|c} & B_1 & B_2 & \text{row min} \\ \hline A_1 & 8 & -3 & -3 \\ A_2 & -3 & 1 & -3 \\ \hline \text{col max} & 8 & 1 & \end{array}$$

min = {max} = 1

max = {min} = -3

minmax ≠ maxmin No saddle point.

Step 2: $p_1 = \frac{d-c}{(a+d)-(b+c)}$

$$= \frac{1-(-3)}{(8+1)-(-3-3)} = \frac{4}{9+6} = \frac{4}{15}$$

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$p_2 = 1 - p_1$

$= 1 - \frac{4}{15}$

$= \frac{15-4}{15} = \frac{11}{15}$

$q_1 = \frac{d-b}{(a+d)-(b+c)} = \frac{4}{15}$

$q_2 = 1 - q_1 = \frac{11}{15}$

$A = \left(\frac{4}{15}, \frac{11}{15}\right) \quad B = \left(\frac{4}{15}, \frac{11}{15}\right)$

$V = \frac{ad-bc}{(a+d)-(b+c)} = \frac{(8\times1)-(-3\times-3)}{15}$

$= \frac{8-9}{15} = \frac{-1}{15}$ //.

Note: If the value is positive. It is advantage to player A.
If the value is negative It is advantage to player B.

2) Determine optimal strategies and value of the game

$$\begin{array}{cc} & B \\ A & \begin{bmatrix} 5 & 1 \\ 3 & 4 \end{bmatrix} \end{array}$$

⇒ Step1: Check for saddle point

$$\begin{array}{ccc} & & \text{row min} \\ & \begin{bmatrix} 5 & 1 \\ 3 & 4 \end{bmatrix} & \begin{array}{c} 1 \\ 3 \end{array} \\ \text{col max} & \begin{array}{cc} 5 & 4 \end{array} & \end{array}$$

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$\min\{\max\} = 4$
$\max\{\min\} = 3$
minmax $\neq$ maxmin No saddle point

Step 2: $p_1 = \dfrac{d-c}{(a+d)-(b+c)} = \dfrac{4-3}{(9)-(4)} = \dfrac{1}{5}$

$p_2 = 1 - p_1 = 1 - \dfrac{1}{5} = \dfrac{4}{5}$

$q_1 = \dfrac{d-b}{(a+d)-(b+c)} = \dfrac{3}{5}$

$q_2 = 1 - q_1 = 1 - \dfrac{3}{5} = \dfrac{2}{5}$

$A = \left(\dfrac{1}{5}, \dfrac{4}{5}\right)$ $B = \left(\dfrac{3}{5}, \dfrac{2}{5}\right)$

$V = \dfrac{ad-bc}{(a+d)-(b+c)} = \dfrac{20-3}{5} = \dfrac{17}{5}$ //

$\therefore$ Strategy advantage is for A.

3) Determine mixed strategies and value of the game

$$\begin{array}{cc} & B \\ A & \begin{bmatrix} 4 & -4 \\ -4 & 4 \end{bmatrix} \end{array}$$

$\Rightarrow$ Step 1: Check for saddle point

$$\begin{array}{cc} \begin{bmatrix} 4 & -4 \\ -4 & 4 \end{bmatrix} & \begin{array}{c} \text{row min} \\ -4 \\ -4 \end{array} \\ \text{col max} \quad 4 \quad\quad 4 & \end{array}$$

$\min\{\max\} = 4$
$\max\{\min\} = -4$
minmax $\neq$ maxmin No saddle point.

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Step 2:

$$p_1 = \frac{d-c}{(a+d)-(b+c)} = \frac{4-(-4)}{(8)-(-8)} = \frac{8}{16} = \frac{1}{2}$$

$$p_2 = 1 - p_1 = 1 - \frac{1}{2} = \frac{1}{2}$$

$$q_1 = \frac{d-b}{(a+d)-(b+c)} = \frac{8}{16} = \frac{1}{2}$$

$$q_2 = 1 - q_1 = 1 - \frac{1}{2} = \frac{1}{2}$$

$$A = \left(\frac{1}{2}, \frac{1}{2}\right) \qquad B = \left(\frac{1}{2}, \frac{1}{2}\right)$$

$$V = \frac{ad-bc}{(a+d)-(b+c)} = \frac{16-16}{16} = \frac{0}{16} = 0$$

34) In a game of matching coins with 2 players suppose player A wins 1 unit of value when there are two heads, win nothing when there are 2 tail tossing coin and lose of $\frac{1}{2}$ unit of value when there are 1 head and 1 tail. determine the pay off matrix, the best strategies for each player and value of the game.

$\Rightarrow$

$$\begin{array}{c c} & \begin{array}{cc} H & T \end{array} \\ \begin{array}{c} H \\ T \end{array} & \begin{bmatrix} 1 & -1/2 \\ -1/2 & 0 \end{bmatrix} \end{array}$$

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Step 1: Check for saddle point

$$\begin{array}{cc} \begin{bmatrix} 1 & -1/2 \\ -1/2 & 0 \end{bmatrix} & \begin{array}{c} \text{row min} \\ -1/2 \\ -1/2 \end{array} \end{array}$$

col max $\quad 1 \quad 0$

min{max} = 0

max{min} = -1/2 $\qquad$ No saddle point

Step 2: $p_1 = \dfrac{d-c}{(a+d)-(b+c)} = \dfrac{0-(-1/2)}{(1+0)-(-\frac{1}{2}-\frac{1}{2})} = \dfrac{1/2}{1+1}$

$= \dfrac{1}{2\times 2} = \dfrac{1}{4}$

$p_2 = 1-p_1 = 1-\dfrac{1}{4} = \dfrac{3}{4}$

$q_1 = \dfrac{d-b}{(a+d)-(b+c)} = \dfrac{1/2}{2} = \dfrac{1}{4}$

$q_2 = 1-q_1 = 1-\dfrac{1}{4} = \dfrac{3}{4}$

$A = \left(\dfrac{1}{4}, \dfrac{3}{4}\right) \qquad B = \left(\dfrac{1}{4}, \dfrac{3}{4}\right)$

$V = \dfrac{ad-bc}{(a+d)-(b+c)} = \dfrac{0-(-1/2 \times -1/2)}{2}$

$= \dfrac{-\frac{1}{4}}{2} = -\dfrac{1}{8}$

Strategy advantage for B.

5) Find value of the game

$$\begin{array}{cc} & B \\ A & \begin{bmatrix} 6 & -3 \\ -3 & 3 \end{bmatrix} \end{array}$$

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Step 1: Check for saddle point

$$\begin{array}{c c} \begin{bmatrix} 6 & -1 \\ -3 & 3 \end{bmatrix} & \begin{array}{c} \text{row min} \\ -3 \\ -3 \end{array} \\ \text{col max} \quad 6 \quad 3 & \end{array}$$

min{max} = 3

max{min} = -1 minmax ≠ maxmin

∴ No saddle point

Step 2:

$$p_1 = \frac{d-c}{(a+d)-(b+c)} = \frac{3-(-3)}{(9)-(-6)} = \frac{6}{15}$$

$$p_2 = 1 - p_1 = 1 - \frac{6}{15} = \frac{9}{15}$$

$$q_1 = \frac{d-b}{(a+d)-(b+c)} = \frac{3-(-1)}{15} = \frac{6}{15}$$

$$q_2 = 1 - q_1 = \frac{9}{15}$$

$$A = \left(\frac{\cancel{6}\,2}{\cancel{15}\,5}, \frac{\cancel{9}\,3}{\cancel{15}\,5}\right) \qquad B = \left(\frac{\cancel{6}\,2}{\cancel{15}\,5}, \frac{\cancel{9}\,3}{\cancel{15}\,5}\right)$$

$$v = \frac{ad-bc}{(a+d)-(b+c)} = \frac{18-9}{15} = \frac{\cancel{9}\,3}{\cancel{15}\,5} = \frac{3}{5}$$

Strategy advantage is for A

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Graphical method for 2×n & m×2 matrix

1)

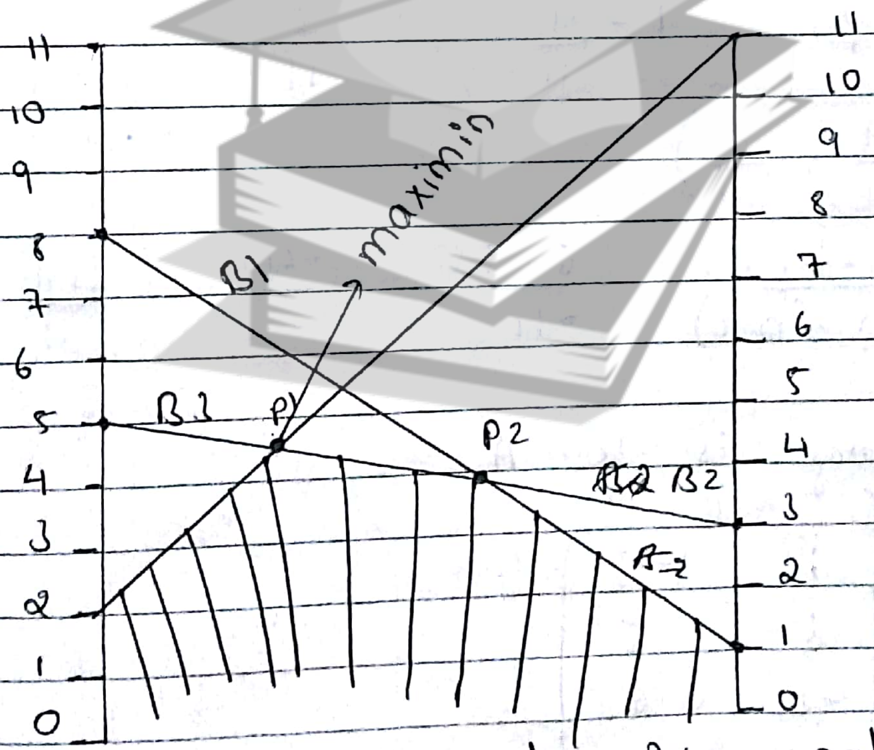
$$A\begin{array}{c}B\\ \begin{bmatrix}1 & 3 & 11\\ 8 & 5 & 2\end{bmatrix}\end{array}$$

	B_1	B_2	B_3	row min
A_1	1	3	11	1
A_2	8	5	2	2
col max	8	5	11	

min{max} = 5

max{min} = 2

minmax ≠ maxmin. No saddle point.



To find maximin for 2×n matrix. Mark the region below the intersection points and find the maximum point.
The 2 intersection points are P1 and P2 and P1 is the maximum, which corresponds to the column B2 and B3

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Consider B3 and B2.

$$\begin{bmatrix} 3 & 11 \\ 5 & 2 \end{bmatrix}$$

$$p_1 = \frac{d-c}{(a+d)-(b+c)} \quad \frac{2-5}{5-16} = \frac{-3}{-11} = \frac{3}{11}$$

$$p_2 = 1 - p_1 = 1 - \frac{3}{11} \quad \frac{8}{11}$$

$$A = \left(\frac{3}{11}, \frac{8}{11}\right)$$

$$q_1 = \frac{d-b}{(a+d)-(b+c)} = \frac{2-11}{-11} = \frac{-9}{-11} = \frac{9}{11}$$

$$q_2 = 1 - p_1 = 1 - \frac{9}{11} = \frac{2}{11}$$

$$B = \left(\frac{9}{11}, \frac{2}{11}\right)$$

$$v = \frac{ad-bc}{(a+d)-(b+c)} = \frac{6-55}{-11} = \frac{-49}{-11} = \frac{49}{11}$$

Advantage is for A.

2)

$$\begin{array}{cc} & B \\ A & \begin{bmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 6 \\ 4 & 1 \\ 2 & 2 \\ -5 & 0 \end{bmatrix} \end{array}$$

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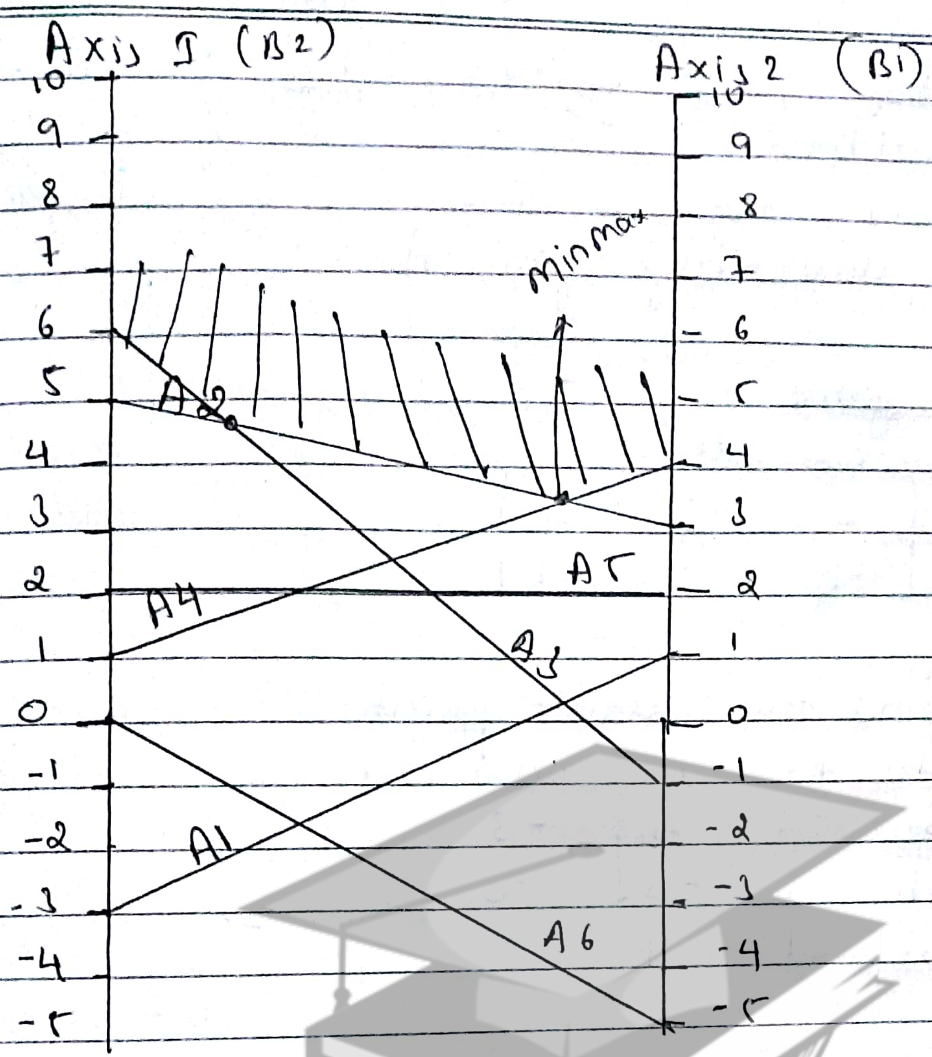

The minmax point is A 4 and A 2

$$\begin{array}{cc} & B \\ A & \begin{bmatrix} 3 & 5 \\ 4 & 1 \end{bmatrix} \end{array}$$

$$p_1 = \frac{d-c}{(a+d)-(b+c)} = \frac{1-4}{4-9} = \frac{-3}{-5} = \frac{3}{5}$$

$$p_2 = 1 - p_1 = 1 - \frac{3}{5} = \frac{5-3}{5} = \frac{2}{5}$$

$$q_1 = \frac{d-b}{(a+d)-(b+c)} = \frac{-4}{-5} = \frac{4}{5}$$

$$q_2 = 1 - q_1 = 1 - \frac{4}{5} = \frac{5-4}{5} = \frac{1}{5}$$

$$A = \left(\frac{3}{5}, \frac{2}{5}\right) \qquad B = \left(\frac{4}{5}, \frac{1}{5}\right)$$

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$V = \frac{ad - bc}{(a+d) - (b+c)} = \frac{3 - 20}{-5} = \frac{17}{5}$ //

Strategy advantage for A

3) For the game

$$\begin{array}{cc} & B \\ A & \begin{bmatrix} 3 & -3 & 4 \\ -1 & 1 & -3 \end{bmatrix} \end{array}$$

Step 1 : Find the saddle point.

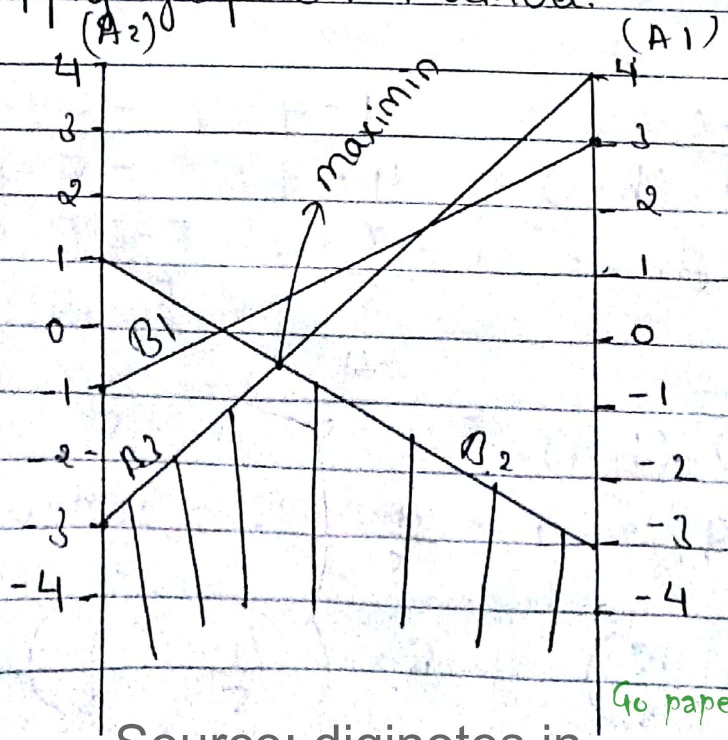
	B1	B2	B3	row min
A1	3	-3	4	-3
A2	-1	1	-3	-3
colmax	3	1	4	

min {max} = -3

max {min} = 1

minmax ≠ maxmin No saddle point

Step 2 : Apply graphical method.



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The intersecting lines are B_2 and B_3

$$\begin{array}{cc} a & b \\ \end{array}$$
$$\begin{bmatrix} -3 & 4 \\ 1 & -3 \end{bmatrix}$$

$$p_1 = \frac{d-c}{(a+d)-(b+c)} = \frac{-3-1}{(-6)-(5)} = \frac{-4}{-11} = \frac{4}{11}$$

$$p_2 = 1 - p_1 = 1 - \frac{4}{11} = \frac{7}{11}$$

$$q_1 = \frac{d-b}{(a+d)-(b+c)} = \frac{-3-4}{-11} = \frac{-7}{-11} = \frac{7}{11}$$

$$q_2 = 1 - \frac{7}{11} = \frac{4}{11}$$

$$A\left(\frac{4}{11}, \frac{7}{11}\right) \qquad B\left(\frac{7}{11}, \frac{4}{11}\right)$$

$$v = \frac{ad-bc}{(a+d)-(b+c)} = \frac{9-4}{-11} = \frac{-5}{11} \;//$$

4)

$$\begin{array}{cc} & B \\ A & \begin{bmatrix} -6 & 7 \\ 4 & -5 \\ -1 & -2 \\ -2 & 5 \\ 7 & 6 \end{bmatrix} \end{array}$$

Step 1: Find out saddle point

			row min
	-6	7	-6
	4	-5	-5
	-1	-2	-2
	-2	5	-2
	7	6	6.
col max	7	7	

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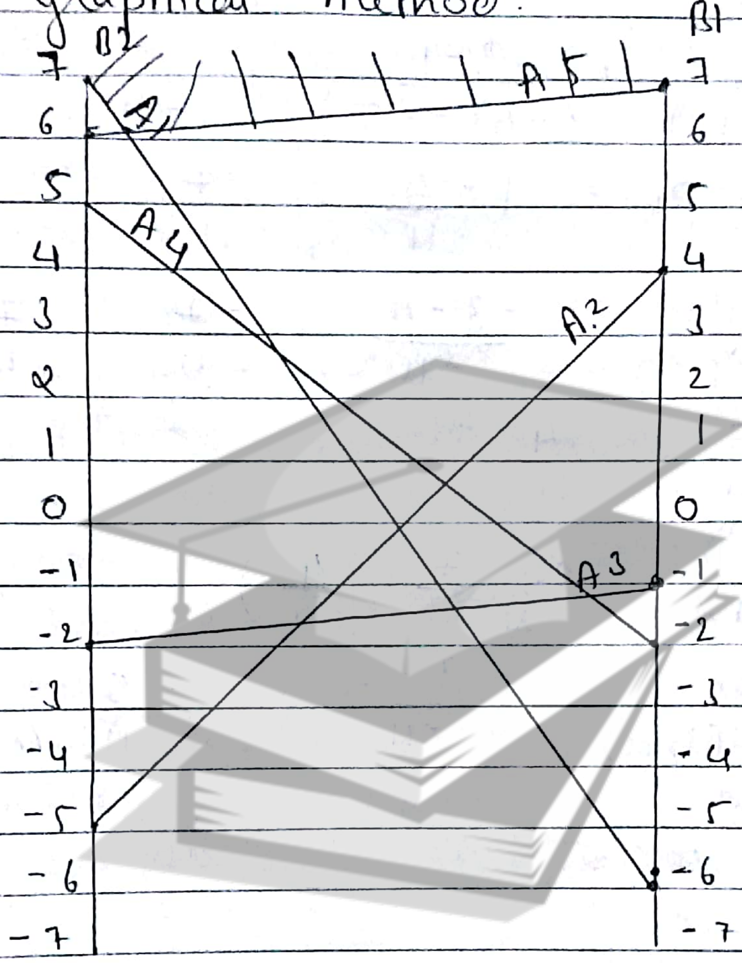
$\max\{\min\} = 7$

$\min\{\max\} = 6$

$\max\{\min\} \neq$ ~~maxmin~~ minmax No saddle point.

Step 2: Graphical method.



Insertion lines are A1 and A5

$$\begin{bmatrix} -6 & 7 \\ 7 & 6 \end{bmatrix}$$

$$p_1 = \frac{d-c}{(a+d)-(b+c)} = \frac{6-7}{(0)-(14)} = \frac{-1}{-14} = \frac{1}{14}$$

$$p_2 = 1 - p_1 = 1 - \frac{1}{14} = \frac{13}{14}$$

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$q_1 = \frac{d-b}{(a+d)-(b+c)}$, $\frac{6-7}{-14} = \frac{1}{14}$

$q_2 = \frac{13}{14}$

$A\left(\frac{1}{14}, \frac{13}{14}\right)$ $B\left(\frac{1}{14}, \frac{13}{14}\right)$

$\nu = \frac{ad-bc}{(a+d)-(b+c)}$, $\frac{(-36)-49}{-14} = \frac{85}{14}$

5. $\begin{bmatrix} 1 & 3 & -3 & 7 \\ 2 & 5 & 4 & -6 \end{bmatrix}$

Step 1: Find saddle point

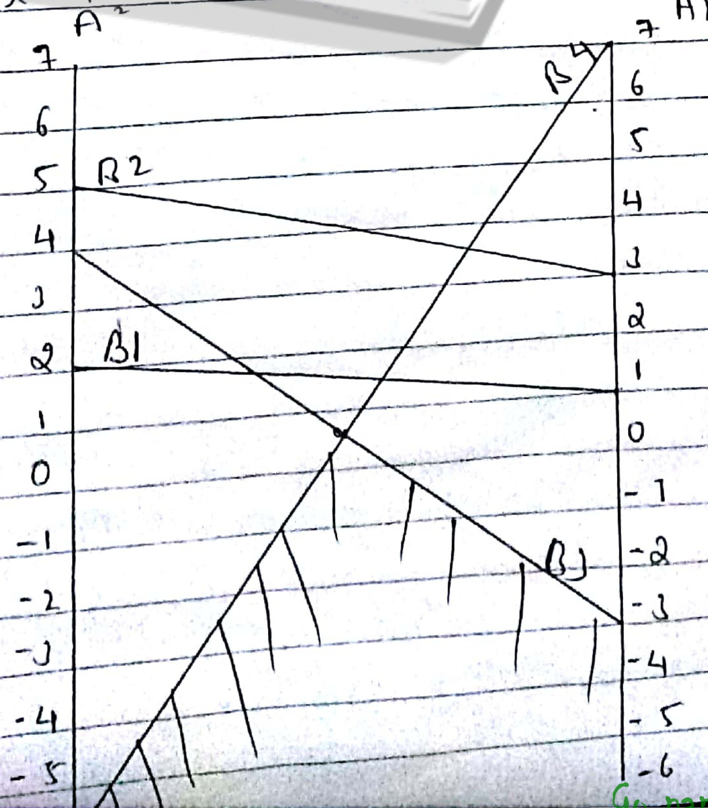
$$\begin{array}{lc} & \text{row min} \\ \begin{bmatrix} 1 & 3 & -3 & 7 \\ 2 & 5 & 4 & -6 \end{bmatrix} & \begin{array}{c} -3 \\ -6 \end{array} \end{array}$$

col max 2 5 4 7

min{max} = -3

max{min} = 2

minmax $\neq$ maxmin No saddle point



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Intersecting lines are B1 and B4

$$\begin{bmatrix} -3 & 7 \\ 4 & -6 \end{bmatrix}$$

$$p_1 = \frac{d-c}{(a+d)-(b+c)} = \frac{-6-4}{(-3-6)-(\cancel{4}7+4)} = \frac{-10}{-9-11} = \frac{10}{20} = \frac{1}{2}$$

$$p_2 = 1 - p_1 = 1 - \frac{1}{2} = \frac{1}{2}$$

$$q_1 = \frac{d-b}{(a+d)-(b+c)} = \frac{-13}{-20} = \frac{13}{20}$$

$$q_2 = 1 - q_1 = 1 - \frac{13}{20} = \frac{7}{20}$$

$$v = \frac{ad-bc}{(a+d)-(b+c)} = \frac{+18-28}{-20} = \frac{-\cancel{46}}{-\cancel{20}} = \frac{10}{20} = \frac{1}{2}$$

Dominance Property:

We use the following rules to reduce a given matrix to a 2x2 matrix or 1x1 matrix

Rule 1: If all the elements in ith row are less than or equal to the corresponding elements ~~of~~ of the jth row, we say that jth strategy dominates ith strategy and hence we delete ith row

$R_i \leq R_j$ delete R_i

Rule 2: If all the elements of the nth column are greater than or equals corresponding elements of the mth column than we say that mth strategy dominates nth strategy hence we ~~go~~ delete nth strategy.

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Rule 3: If row dominance and column dominance cannot reduce a matrix then we take average

1. If all the elements of the ith row less than or equals the average of two or more rows than we say that the group of rows dominates ith row. Hence we delete ith row.

2. If all the elements of the nth column are greater than or equals the average of two or more columns than we say that group of columns dominant nth column. Hence we delete nth column.

1) Solve the game by applying dominance

	b_1	b_2	b_3
a_1	5	20	-10
a_2	10	6	2
a_3	20	15	18

	b_1	b_2	b_3
a_1	5	20	-10
~~a_2~~	~~10~~	~~6~~	~~2~~
a_3	20	15	18

Compare all possible combinations of rows.

Step 1: $a_2 \leq a_3$ delete a_2

Step 2: Compare all possible combinations of column

$b_3 \leq b_1$ b_3 dominates b_1

delete b_1

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$$\begin{bmatrix} 20 & -10 \\ 15 & 18 \end{bmatrix} \quad \begin{array}{c} \text{row min} \\ -10 \\ 15 \end{array}$$

col max 20 18

min{max} = 15

max{min} = 18

minmax ≠ maxmin No saddle point

$$\begin{bmatrix} 20 & -10 \\ 15 & 18 \end{bmatrix}$$

$$p_1 = \frac{d-c}{(a+d)-(b+c)} = \frac{18-15}{(38)-(5)} = \frac{3}{33} = \frac{1}{11}$$

$$p_2 = 1 - p_1 = 1 - \frac{1}{11} = \frac{10}{11}$$

$$q_1 = \frac{d-b}{(a+d)-(b+c)} = \frac{18+10}{33} = \frac{28}{33}$$

$$q_2 = 1 - q_1 = 1 - \frac{28}{33} = \frac{\cancel{33}}{\cancel{33}} \quad \frac{5}{33}$$

$$V = \frac{ad-bc}{(a+d)-(b+c)} = \frac{360+150}{33} = \frac{510}{33}$$

Stratgy advantage for game A

2)

	b1	b2	b3	b4	b5
a1	2	4	3	8	4
a2	5	6	3	7	8
a3	6	7	9	8	7
a4	4	2	8	4	3

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$\Rightarrow$

	b_1	b_2	b_3	b_4	b_5
a_1	2	4	3	8	4
a_2	5	6	3	7	8
a_3	6	7	9	8	7
a_4	4	2	8	4	3

$a_1 \le a_3$ a_1 is deleted. , a_1 is dominating

$a_4 \le a_3$ a_4 is deleted. , a_4 is dominating

$b_1 \le b_2$ b_2 is deleted b_2 is dominating

$b_1 \le b_5$ b_5 is deleted b_5 is dominating.

~~b_4~~ $b_1 \le b_4$ b_4 is deleted b_4 is dominating.

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$a_2 \le a_3$, a_2 is deleted a_3 is dominating
$b_1 \le b_3$ b_3 is deleted b_3 is dominating.

$\boxed{v = 6}$

3)

	b_1	b_2	b_3
a_1	1	2	0
a_2	2	-3	-2
a_3	0	3	-1

⇒

	b_1	b_2	b_3
a_1	1	2	0
a_2	2	-3	-2
a_3	0	3	-1

$b_1 > b_3$ b_1 is deleted
$a_2 < a_1$ a_2 is deleted
$b_2 > b_3$ b_2 is deleted
$a_3 \le a_1$ a_3 is deleted.

Value of the game is 0.

4) Solve the game using dominance property

$$\begin{array}{c|cccc} & b_1 & b_2 & b_3 & b_4 \\ a_1 & 2 & -2 & 4 & 1 \\ a_2 & 6 & 1 & 12 & 3 \\ a_3 & -3 & 2 & 0 & 6 \\ a_4 & 2 & -3 & 7 & 7 \end{array}$$

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$$\begin{array}{c|cccc} & b_1 & b_2 & b_3 & b_4 \\ \hline a_1 & \cancel{2} & \cancel{-2} & \cancel{1} & \\ a_2 & 6 & 1 & \cancel{12} & \cancel{3} \\ a_3 & -3 & 2 & \cancel{0} & \cancel{6} \\ a_4 & \cancel{2} & \cancel{-3} & \cancel{7} & \cancel{7} \end{array}$$

$a_1 < a_2$ delete a_1

$b_4 > b_2$ delete b_4

$b_3 > b_1$ delete b_3

$a_2 \geq a_4$ delete a_4

$$\begin{bmatrix} 6 & 1 \\ -3 & 2 \end{bmatrix} \quad \begin{array}{c} \text{row min} \\ 1 \\ -3 \end{array}$$

col max $\quad 6 \quad 2$

$\min\{\max\} = 2$

$\max\{\min\} = 1$

maxmin $\neq$ min max No saddle point.

$$p_1 = \frac{d-c}{(a+d)-(b+c)} = \frac{2-(-3)}{(6+2)-(1-3)} = \frac{5}{8+2} = \frac{5}{10} = \frac{1}{2}$$

$$p_2 = 1 - p_1 = 1 - \frac{1}{2} = \frac{1}{2} \qquad A\left(\frac{1}{2}, \frac{1}{2}\right)$$

$$q_1 = \frac{d-b}{(a+d)-(b+c)} = \frac{(2-1)}{10} = \frac{1}{10} \qquad B\left(\frac{1}{10}, \frac{9}{10}\right)$$

$$q_2 = 1 - q_1 = 1 - \frac{1}{10} = \frac{9}{10}$$

$$v = \frac{ad-bc}{(a+d)-(b+c)} = \frac{12+3}{10} = \frac{15}{10} = \frac{3}{2} //$$

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5) The following matrix represents the pay off to P_1 in a rectangular game between two persons P1 and P2 by using dominance property reduce the game to 2×4 and solve it graphically.

$$\begin{array}{cc} & P_2 \\ P_1 & \begin{bmatrix} 8 & 15 & -4 & -2 \\ 19 & 15 & 17 & 16 \\ 0 & 20 & 15 & 5 \end{bmatrix} \end{array}$$

~~R1~~ $a_1 < a_2$ delete a_1

$$\begin{bmatrix} \cancel{8} & \cancel{15} & \cancel{-4} & \cancel{-2} \\ 19 & 15 & 17 & 16 \\ 0 & 20 & 5 & 5 \end{bmatrix}$$

$$\begin{array}{cccccc} & b_1 & b_2 & b_3 & b_4 & \text{rowmin} \\ a_1 & 19 & 15 & 17 & 16 & 15 \\ a_2 & 0 & 20 & 5 & 5 & 0 \\ \text{colmax} & 0 & 15 & 5 & 5 & \end{array}$$

min{max} = 0
max{min} = 15
minmax $\neq$ maxmin No saddle point

Apply graphical method.

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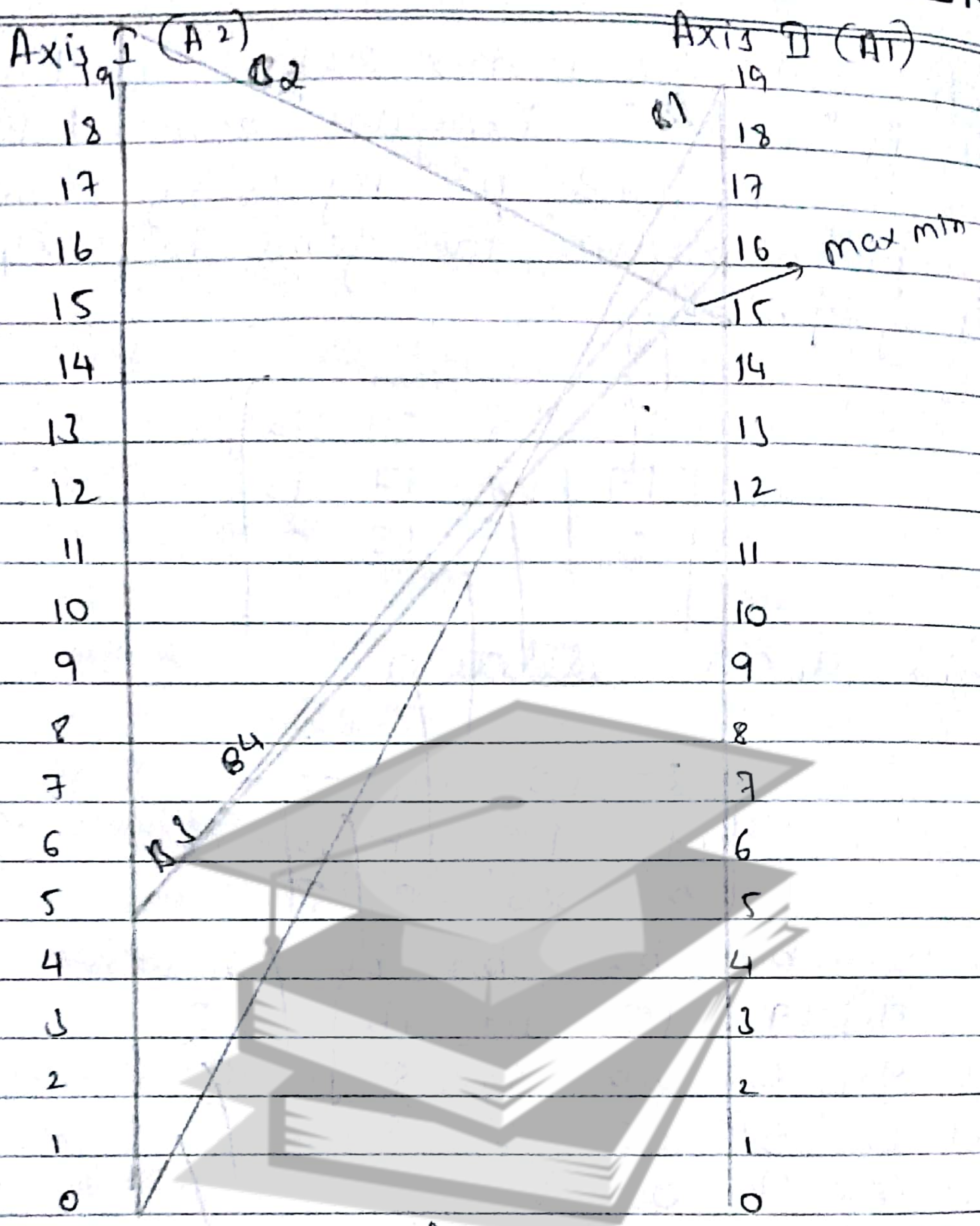
6)



The lines are B2, B4

$$\begin{bmatrix} 15 & 16 \\ 20 & 5 \end{bmatrix}$$

$$P_1 = \frac{d-c}{(a+d)-(b+c)} = \frac{5-20}{(15+5)-(16+20)} = \frac{-15}{20-36} = \frac{15}{16}$$

$$P_2 = 1 - P_1 = 1 - \frac{15}{16} = \frac{1}{16}$$

$$q_1 = \frac{d-b}{(a+d)-(b+c)} = \frac{5-16}{-16} = \frac{-11}{-16} = \frac{11}{16}$$

$$q_2 = 1 - q_1 = 1 - \frac{11}{-16} \quad \frac{5}{16}$$

$$V = \frac{ad-bc}{(a+d)-(b+c)} = (15\times5)-(16\times20) \,|\, -16 = \frac{245}{16}$$

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